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SPONTANEOUS CONDENSATION OF NITROGEN IN A FLAT NOZZLE IN A CRYOGENIC  
WIND TUNNEL

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Experimental [1] and numerical [2-4] studies of the spontaneous condensation of water vapor in two-dimensional nozzles demonstrate the significant effect of the three-dimensional nature of flow on the configuration of the phase transition zone. It was observed that, depending on the specific conditions, oblique (with a positive or negative slope), arched, locally-shaped, or other-shaped condensation jumps occur. In regard to wind tunnels with a slightly condensed flow in their working part, such phenomena can be an additional source of perturbations of the theoretical flow field. In turn, besides the feature just noted, in transonic cryogenic wind tunnels it can be expected that the bidimensionality will also affect the oscillatory state of the flow. As is known [4], such a state is realized with the occurrence of a condensation jump in the region of moderate supersonic values of the Mach number. As a result, there may be a change in the zones of existence of steady flow, the boundaries of which in a unidimensional formulation were determined in [5].

In connection with the above, it is of practical interest to analyze features of the occurrence of oscillatory flow in a flat nozzle, the contour of which models a shaped nozzle in a cryogenic wind tunnel. Here, the modification of the method of S. K. Godunov developed in [3] is an effective tool for numerical study of the nonsteady interaction of different types of wave structures in transonic flows.

1. We will examine a two-dimensional flow of a spontaneously condensing gas. Here, we make the usual assumptions for this case, to wit: The system is adiabatic; the flow as a whole may be steady or nonsteady; phase slip is absent; the condensing gas is thermally and calorically ideal; the process of core formation occurs in a quasisteady manner; the condensate is

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uniformly distributed in the gas phase; the drops are spherical and the rate of change in their radius is independent of their size; both condensation and evaporation are possible. It is assumed that the drop flow regime can be characterized as free-molecular and that drop growth rate is determined by the Knudsen formula.

As a result, the dynamic equations for the medium as a whole and the kinetic equations appear as follows in divergent form

$$\frac{\partial ay^{v-1}}{\partial t} + \frac{\partial by^{v-1}}{\partial x} + \frac{\partial cy^{v-1}}{\partial y} = f, \quad (1.1)$$

$$a = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho \left( e + \frac{w^2}{2} \right) \end{bmatrix}, \quad b = \begin{bmatrix} \rho u \\ p + \rho u^2 \\ \rho uv \\ \rho u \left( e + \frac{p}{\rho} + \frac{w^2}{2} \right) \end{bmatrix},$$

$$c = \begin{bmatrix} \rho v \\ \rho uv \\ p + \rho v^2 \\ \rho v \left( e + \frac{p}{\rho} + \frac{w^2}{2} \right) \end{bmatrix}, \quad f = \begin{bmatrix} 0 \\ p(v-1) \\ 0 \\ 0 \end{bmatrix},$$

$$e = e_1\beta + e_2(1-\beta), \quad e_1 = \frac{1}{\kappa-1} \frac{p}{\rho_1} + \xi_1, \quad e_2 = c_2 T_s + \xi_2,$$

$$p = \rho_1 R T, \quad \rho = \rho_1 \beta^{-1},$$

$$\frac{\partial \beta \rho y^{v-1}}{\partial t} + \frac{\partial \beta \rho u y^{v-1}}{\partial x} + \frac{\partial \beta \rho v y^{v-1}}{\partial y} = -\rho \omega y^{v-1},$$

$$\frac{\partial \rho \Omega_i y^{v-1}}{\partial t} + \frac{\partial \rho \Omega_i u y^{v-1}}{\partial x} + \frac{\partial \rho \Omega_i v y^{v-1}}{\partial y} = \rho \omega_i y^{v-1},$$

$$\omega = 4\pi \rho_2 \left( \dot{r} \Omega_2 + \frac{1}{3} \frac{J}{\rho} r_*^3 \right), \quad \omega_i = i \dot{r} \Omega_{i-1} + \frac{J}{\rho} r_*^i,$$

$$\Omega_i = \int_{r_*}^{\infty} r^i f(r) dr, \quad i = 0, 1, 2,$$

where  $p$  is pressure;  $\rho$  is the density of the mixture;  $\rho_1$  is the density of the gas phase;  $T$  is its temperature;  $T_s$  is the saturation temperature;  $u$  and  $v$  are projections of the velocity vector  $w$  onto the  $x$  and  $y$  axes of the Cartesian coordinate system;  $e$ ,  $e_1$ , and  $e_2$  are the internal energy of the mixture, gas phase, and liquid phase, respectively;  $\kappa$  is the adiabatic exponent;  $R$  is the gas constant;  $\beta$  is the mass concentration of the gas phase;  $\xi_{1,2}$  are constants the values of which are determined by the thermophysical properties of the substance and the reading system;  $J$  is the rate of core formation, determined from the Frenkel-Zel'dovich formula;  $r$  is the drop radius;  $r_*$  is the radius of the condensation core;  $\dot{r}$  is the rate of drop growth;  $v = 1, 2$  for planar and axisymmetric flows, respectively.

A difference approximation of Eqs. (1.1) for integration by the method of S. K. Godunov is constructed by a familiar procedure [4]. The same work presents recommendations on realizing a numerical algorithm.

2. Let us examine the results of calculation of nitrogen flows in a flat model nozzle for  $M = 1.29$ . The contour of the nozzle was prescribed by analogy with [5] and is shown in Fig. 1. Here, the size of the critical section  $h_* = 10$  cm, the length of the subsonic part was 70 cm, and the length of the supersonic part was 55 cm. The number of cells in the subdivision was 100 over the longitudinal coordinate  $x$  and 9 over the vertical coordinate, which corresponds to the maximum capabilities of the BESM-6 computer for solving the given problem. The thermophysical parameters of nitrogen were taken from tables in [6].

First we compared results of calculation of steady flow in one- and two-dimensional formulations with different numbers of cells over the coordinate  $y$ . The corresponding data for  $p_0 = 7 \cdot 10^5$  Pa and  $T_0 = 104^\circ\text{K}$  is shown in Fig. 2 in the form of nozzle-axis distributions of isobaric supercooling  $\Delta T$ , K (curves I) and the ratio of the static temperature  $T$  to the temperature  $T_0$  (curves II).

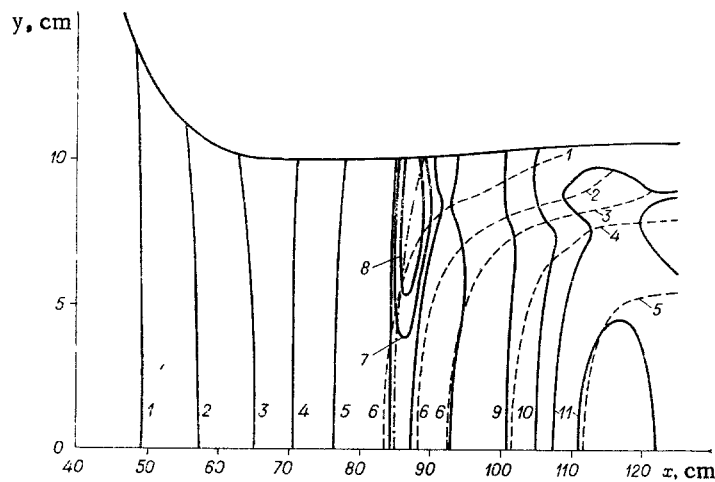


Fig. 1

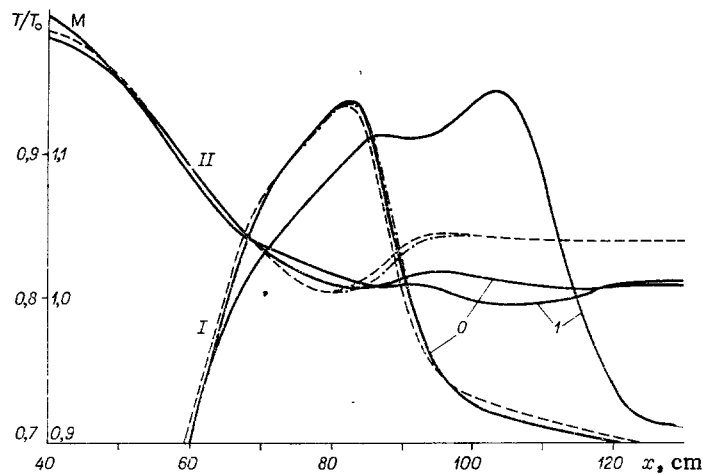


Fig. 2

The dashed curve shows results of calculation with a unidimensional formulation [5], while the dot-dashed line shows results in the two-dimensional formulation for the axial part of the flow limited to three cells over the  $y$  axis. The solid line shows results of calculation in a two-dimensional formulation for a full-profile nozzle with nine cells over the transverse axis. The zero denotes curves corresponding to the nozzle axis, while the number one corresponds to the nozzle wall.

Comparison of the dashed and dot-dash curves shows that for "unidimensional" nozzles the calculation in the two-dimensional approximation is practically identical to the calculation in the one-dimensional approximation.

Proceeding to analysis of the effect of spatiality (three dimensions), we first note that nearly the same distribution of supercooling  $\Delta T$  is obtained on the axis of the two-dimensional nozzle as in the unidimensional approximation.

This finding agrees with the corresponding conclusion made in [7], for example. As a result, in the one- and two-dimensional variants the same number of condensation centers is formed on the nozzle axis, and the phase transition rates and quantities of heat liberated by vaporization are nearly the same.

The static-temperature distributions on the nozzle axis agree relatively well up to the Wilson point. After this point, nonequilibrium condensation has a considerably smaller effect in the two-dimensional approximation than in the unidimensional approximation. There is the same degree of difference in the distributions on the nozzle wall compared to the unidimensional distribution and the distributions on the nozzle axis. Here, the distribution of supercooling on the wall has two local maximums.

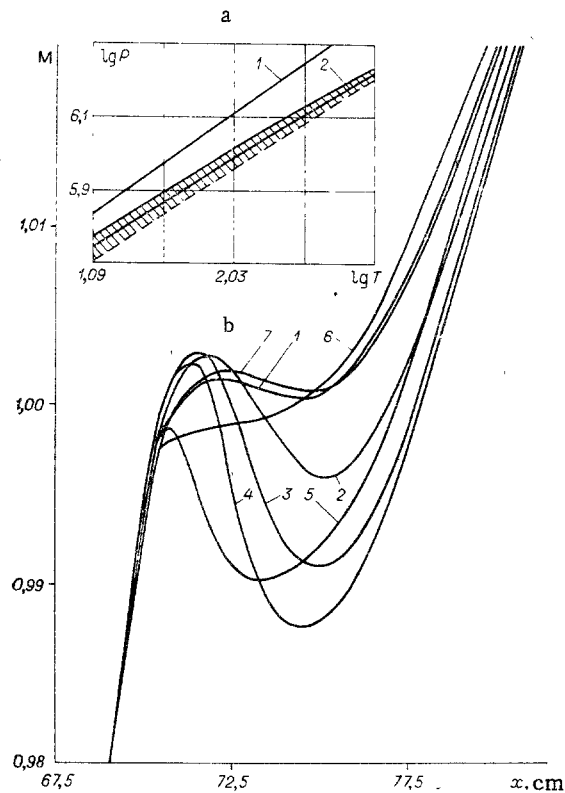


Fig. 3

The effect of bidimensionality can be explained on the basis of analysis of the parameter fields shown in Fig. 1. Here we have plotted lines of equal values of  $M$  (solid curves 1-11,  $M = 0.5, 0.7, 0.9, 1.0, 1.05, 1.1, 1.102, 1.104, 1.13, 1.15, 1.17$ , respectively); lines of equal values of the degree of condensation  $\delta = 1 - \beta$  (dashed curves 1-5,  $\delta = 0.1, 1, 2, 3, 2.5\%$ , respectively); line of maximum supercooling (dot-dash line).

In accordance with the shape of the lines of equal values of  $M$ , in the supersonic part of the nozzle the maximum supercooling first occurs on the nozzle axis, and condensation begins here (before it begins on the wall). The perturbations generated by condensation propagate from the nozzle axis along the Mach lines to the wall and back. Here, the zone of the drop in  $M$  at the wall in the region of the coordinate  $x = 90$  cm corresponds almost exactly to the Mach line with  $M = 1.1$ , which begins on the condensation front on the nozzle axis at  $x = 85$  cm. As a result, there is an increase in pressure and temperature at the wall in this region, supercooling diminishes (see Fig. 2), and the beginning of condensation is delayed.

Since the effect of the phase transition on the flow parameters is concentrated in the axial part, the flow may expand toward the periphery. This is the reason for the slight perturbation of the gasdynamic parameters in the condensation jump in the two-dimensional flow compared to the case of the unidimensional approximation.

3. Calculations performed in a unidimensional formulation show the successive realization of three types of oscillatory flow states with a decrease in the stagnation temperature [4]. The first state is when the zone of movement of the nonsteady shock wave is located entirely within the supersonic part of the nozzle. If this zone embraces the critical section, then the second type of oscillatory state is realized. The third type of oscillatory state is characterized by pulsations of the subsonic flow.

Calculations were performed in the two-dimensional formulation in the range  $p_0 = 5 \cdot 10^5 - 15 \cdot 10^5$  Pa,  $T_0 = 98 - 116^\circ\text{K}$ . They indicate the presence only of the second type of free oscillations. Meanwhile, the flow stabilizes with both an increase and a decrease in the stagnation temperature. Thus, the nitrogen saturation line 1 is plotted in the coordinates  $\log p - \log T$  in Fig. 3a. The dashed line is the boundary of the region of stagnation parameters  $p_0$  and  $T_0$ , the crossing of which signifies the realization of the first type of oscillatory state. The solid line 2 is the boundary of the region of  $p_0$  and  $T_0$  for the third type of oscillatory state for a unidimensional flow [5]. The hatched area shows the zone of existence of the oscillatory state in the two-dimensional nozzle.

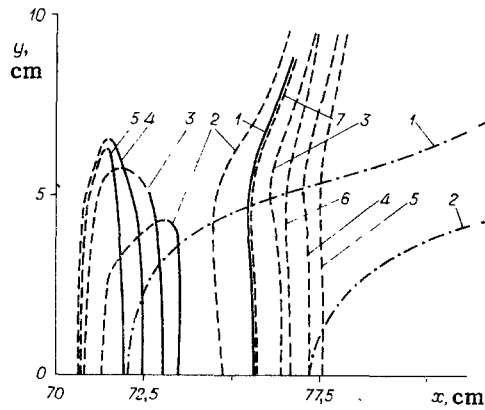


Fig. 4

Figure 3b shows the distributions of  $M$  along the nozzle axis at successive moments of time over one oscillation period beginning with curve 1 and ending with curve 7. In this case,  $p_0 = 15 \cdot 10^5$  Pa,  $T_0 = 115.5^\circ\text{K}$ , and the pulsation frequency is 26 Hz.

Figure 4 shows the evolution of the isolines  $M = 1$  over another oscillation period, which in this case begins with state 1 with a degenerate shock wave and ends with state 7 (corresponding to curve 6 in Fig. 3b). Here, the solid lines 2-5 show the positions of the shock wave, while the dashed lines are lines of the transition through the speed of sound near the critical cross section of the nozzle at  $x = 70$  cm and in the zone of subsequent expansion of the flow. The dot-dash curve 1 represents the isoline  $\delta = 0.1\%$ , while curve 2 represents the isoline  $\delta = 1\%$  for flow state 2.

In the present case, bidimensionality is seen to have a greater effect than in the variant with a stationary condensation jump. Thus, there is an increase in the size of the region of reduced values of  $M$  at the wall, while the condensation zone occupies scarcely half the cross section of the nozzle. There is also some increase in the lifetime of the state with a degenerate shock wave (curves 6, 7).

Thus, initial bidimensionality of the flow leads to a situation whereby condensation begins on the nozzle axis and, affecting the state of the gas at the wall, delays the development of the phase transition from the peripheral part of the flow. This in turn reinforces the effect of bidimensionality on the one hand and, on the other hand, leads to a smoother change in the parameters in the condensation zone and a contraction of the region of existence of the oscillatory flow states.

In conclusion, we should note that the frequency characteristics obtained in the two-dimensional formulation agree well with those given by the unidimensional formulation [5].

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